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Crossover from weak to strong coupling superconductivity in multi-band systems

Francisco Dinóla Neto[1], **Mucio A Continentino**[2] and **Claudine Lacroix**[3]

[1] Instituto de Física, Universidade Federal Fluminense, Campus da Praia Vermelha, Niterói, RJ, 24.210-340, Brazil
[2] Centro Brasileiro de Pesquisas Físicas, Rua Dr Xavier Sigaud, 150-Urca, Rio de Janeiro, RJ, 22290-180, Brazil
[3] Institut Néel, CNRS-UJF, 25 avenue des Martyrs, BP 166, 38042 Grenoble Cedex 9, France

E-mail: claudine.lacroix@grenoble.cnrs.fr

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Abstract
The study of superconductivity in correlated systems is an exciting area of condensed matter physics. In this paper we consider superconducting ground states in systems described by two-band models with different effective masses. These two bands are coupled through an effective hybridization that can be directly tuned by pressure. We consider the cases of s-wave superconductivity associated with the electrons in a narrow band and also with inter-band pairing. To study the system in the strong coupling regime we introduce the s-wave scattering length a_s, and obtain the superconducting order parameters and the chemical potential as functions of the interaction strength $1/k_F a_s$ along the BCS–BEC crossover at $T = 0$. Finally, we discuss the phase diagram of this model as a function of external pressure and how our results can be applied for two-band systems as Fe pnictides or heavy fermions. The main result of this study is the occurrence of a superconducting quantum critical point (SQCP) in this two-band model.

1. Introduction

Superconductivity in the strong coupling regime has been widely studied through simple models, such as in a gas of interacting fermions [1, 2]. These studies become particularly interesting when they include the crossover to this regime from weak coupling BCS superconductivity. In this case one finds a continuous evolution from large Cooper pairs to Bose–Einstein condensation of bound fermions, as the strength of the interaction increases. Although this problem was studied many years ago [2, 3], it has raised new interest as possibly being relevant to understanding high temperature superconductors [4, 5]. In the last decade important progress has been made in understanding the smooth evolution of the transition temperatures [6], the description of the broken-symmetry states along the BCS–BEC crossover [7], and the thermodynamics along the BCS–BEC crossover [9] including the thermodynamic properties of d-wave superconductors [8]. More recently, studies of phase transitions in collections of cold atoms [10] has further increased the interest in this problem, as the weak to strong coupling crossover can be realized experimentally in these systems [2].

The crossover from weak coupling superconductivity to Bose–Einstein condensation occurs as a smooth evolution between two very different physical systems. In conventional BCS superconductors there is a critical temperature T_c where superconductivity is suppressed due to thermal breakdown of Cooper pairs. In the case of a Bose superconductor there are two characteristic temperatures, one is the critical temperature, which in the extreme Bose regime does not depend on the coupling strength and the other, significantly higher, is associated with thermal pair decomposition.

In this work we propose to study the physics of the BCS–BEC crossover and its relevance for condensed matter systems, focusing on a multi-band model, which might be more appropriate for studying systems as Fe pnictides or heavy fermions. We consider a two-band model where the quasi-particles have different effective masses. One is a narrow band of heavy quasi-particles and another a wide band associated with uncorrelated conduction electrons. This model can be used to describe cerium (Ce), ytterbium (Yb), and uranium (U) compounds, where the narrow band is formed by unfilled shells of f-electrons and the wide band describes s, p, or d quasi-particles. For systems such as the Fe pnictides, the narrow band is formed by d-electrons.

We study for completeness a two-band model with intra-band and inter-band attractive interactions and a hybridization term. The origins of these interactions are left unspecified and their relative importance can be varied from system to system. The reason for including hybridization explicitly is that this can be easily tuned by external pressure. Consequently, it can be used as a control parameter to explore the phase diagram of these systems, in particular the existence of superconductor quantum critical points (SQCP) separating normal and superconducting phases. The present model can also be used to describe superconducting states [11, 12] with different types of symmetries [13] in a wide region of the parameter space.

The experimental studies of the solid state systems that we are interested in here differ significantly from those on cold atom systems. In the latter it is possible to control the strength of the interactions [2] while in the former this is not possible in principle. In the solid state systems the most common control parameters are doping, pressure, or magnetic field. In particular, the ratio of the hybridization over the bandwidth can be tuned by external pressure. While it is possible that the strength of the attractive interactions depends on hybridization, its most important effect is to transfer quasi-particles between the different renormalized bands. For intra-band superconductivity, which generally dominates in metals [14], this weakens the effect of the attractive interactions and eventually destroys superconductivity. In some sense the effect we study by changing V has features similar to doping the system, if we could exclude the effect of disorder.

For the purpose of characterizing the superconducting phase, we use the equations of motion method to obtain both normal and anomalous Green functions at zero temperature. We show that in the case of intra-band interactions, the transition from the superconductor to the normal state as hybridization increases is continuous in both weak and strong coupling regimes. The hybridization shifts the chemical potential in the BCS limit, while in the strong coupling regime both the gap and the chemical potential converge to hybridization-independent results. For inter-band attractive interactions we find that the superconductor–normal transition driven by hybridization (pressure) is discontinuous all along the crossover from weak to strong coupling.

The paper is organized as follows: in section 2 we introduce the model and obtain the equations of motion for the Green functions. These are solved yielding all the important propagators. From these propagators we extract the relevant correlation functions, namely the superconductor order parameter and the number density. The latter is essential if we want to investigate the weak to strong coupling crossover. In section 3 we discuss the effects of hybridization on superconductivity in both weak and strong coupling regimes. For this purpose we have to introduce an s-wave scattering length [7]. We discuss the effects of hybridization on inter-band and intra-band s-wave superconductivity [15] all along the BCS–BEC crossover. We obtain for both cases the critical value of hybridization V_c, which is required to destroy superconductivity at zero temperature. This value increases smoothly all along with the strength of the interaction.

2. The model and the Green functions

The two-band model Hamiltonian with hybridization is given by,

$$
\begin{aligned}
\mathcal{H} = {} & \sum_{\mathbf{k},\sigma} \varepsilon^s_{\mathbf{k}} c^\dagger_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma} + \sum_{\mathbf{k},\sigma} \varepsilon^f_{\mathbf{k}} f^\dagger_{\mathbf{k},\sigma} f_{\mathbf{k},\sigma} \\
& + V \sum_{i\sigma} (c^\dagger_{i\sigma} f_{i\sigma} + f^\dagger_{i\sigma} c_{i\sigma}) - U \sum_{i\sigma} n^c_{i\sigma} n^f_{i-\sigma} \\
& - \sum_{ij\sigma} G_{ij} n^f_{i\sigma} n^f_{j-\sigma} - \mu N
\end{aligned} \tag{1}
$$

where V is the hybridization term, U is a local inter-band attraction, G_{ij} is an intersite f–f attraction, and N the total number of lattice sites. The two-band model is used to describe a large variety of systems, such as transition metals [19], cuprate compounds [20], excitonic correlations [21], asymmetric superconductivity [22], heavy fermions [18], and the recently discovered family of superconductors based on FeAs [23]. In fact, a complete description requires us to include local repulsion between f-electrons [24]; however, our purpose here is to point out that the BEC–BCS crossover is different in a two-band case, compared with the one-band case, and this is the reason why we do not take into account explicitly this local f–f repulsion. The parameters in equation (1) should be considered as effective parameters, such as, for example, those obtained for the Kondo lattice in the slave boson method [17] or as in [16].

With the use of a BCS decoupling in equation (1), and performing a Fourier transformation, we obtain the normal and anomalous Green functions for the electrons of the two bands:

$$
\begin{aligned}
\langle\langle f_{k\sigma}; f^\dagger_{k\sigma}\rangle\rangle = {} & \frac{1}{2\pi}[(\omega^2 - \varepsilon^{s2}_k)(\omega + \varepsilon^f_k) - \Delta^2_{\mathrm{cf}}(\omega + \varepsilon^s_k) \\
& - V^2(\omega - \varepsilon^s_k)]P(\omega)^{-1},
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
\langle\langle c_{k\sigma}; c^\dagger_{k\sigma}\rangle\rangle = {} & \frac{1}{2\pi}[(\omega^2 - \varepsilon^{f2}_k)(\omega + \varepsilon^s_k) - \Delta^2_{\mathrm{cf}}(\omega + \varepsilon^f_k) \\
& - V^2(\omega - \varepsilon^f_k) - \Delta^2_k(\omega + \varepsilon^s_k)]P(\omega)^{-1},
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
\langle\langle f^\dagger_{-k-\sigma}; c^\dagger_{k\sigma}\rangle\rangle = {} & \frac{1}{2\pi}[\Delta^3_{\mathrm{cf}} + V\Delta_k(\omega + \varepsilon^s_k) - \Delta_{\mathrm{cf}}V^2 \\
& - \Delta_{\mathrm{cf}}(\omega + \varepsilon^s_k)(\omega - \varepsilon^f_k)]P(\omega)^{-1},
\end{aligned} \tag{4}
$$

$$
\langle\langle f^\dagger_{-k-\sigma}; f^\dagger_{k\sigma}\rangle\rangle = \frac{1}{2\pi}[2\Delta_{\mathrm{cf}}\omega V - \Delta_k(\omega^2 - \varepsilon^{s2}_k)]P(\omega)^{-1}, \tag{5}
$$

with

$$
\begin{aligned}
P(\omega) = {} & \omega^4 - [\varepsilon^{s2}_k + \varepsilon^{f2}_k + 2(\Delta^2_{\mathrm{cf}} + V^2) + \Delta^2_k]\omega^2 \\
& + \{[\varepsilon^s_k\varepsilon^f_k - (\Delta^2_{\mathrm{cf}} - V^2)] + \Delta^2_k\varepsilon^{s2}_k\},
\end{aligned} \tag{6}
$$

where Δ_{cf} and Δ_k are respectively the inter- and intra-band superconductor order parameters, defined as $\Delta_{\mathrm{cf}} = U\langle c^\dagger_{i\sigma} f^\dagger_{i-\sigma}\rangle$ and $\Delta_k = \sum'_k G(k,k')\langle f^\dagger_{k'\sigma} f^\dagger_{-k'-\sigma}\rangle$, where $G(k,k')$ is defined in equation (10) below. The roots of the polynomial $P(\omega)$ determine the excitation energies of the system,

$$\omega_{1,2} = \sqrt{A(k) \pm \sqrt{B(k)}}, \tag{7}$$

with

$$A(k) = \frac{\varepsilon_k^{s2} + \varepsilon_k^{f2} + 2(\Delta_{\rm cf}^2 + V^2) + \Delta_k^2}{2}$$

and

$$B(k) = \left[\frac{\Delta_k^2 - (\varepsilon_k^{s2} - \varepsilon_k^{f2})}{2}\right]^2 + V^2[(\varepsilon_k^s + \varepsilon_k^f)^2 + \Delta_k^2] + \Delta_{\rm cf}^2[(\varepsilon_k^s - \varepsilon_k^f)^2 + \Delta_k^2 + 4V^2].$$

In the following, we assume that the bands are homotetic, i.e. $\varepsilon_k^f = \alpha\varepsilon_k^s$, $\varepsilon_k^s = k^2/2m - \mu$ are the energies of single particle excitations. The bandwidths of the s and f bands are W and D respectively, where $D = \alpha W$. The quantity $\alpha < 1$ is the ratio of the effective masses of the quasi-particles in the two bands. The total number of electrons $N = n_{\rm f} + n_{\rm c}$ is taken as fixed.

3. Hybridization effects

Next, we obtain the number equation and the self-consistent gap equations for both the intra-band and inter-band order parameters. The $T = 0$ number equation is obtained from the propagators (2) and (3). It is given by,

$$N = \sum_k \left\{1 - \frac{1}{2(\omega_1^2 - \omega_2^2)}\left[\frac{(\varepsilon_k^s + \varepsilon_k^f)(\omega_1^2 - \Delta_{\rm cf}^2 + V^2)}{\omega_1} - \frac{\varepsilon_k^s[\Delta_k^2 + \varepsilon_k^f(\varepsilon_k^f + \varepsilon_k^s)]}{\omega_1} - \frac{(\varepsilon_k^s + \varepsilon_k^f)(\omega_2^2 - \Delta_{\rm cf}^2 + V^2)}{\omega_2} + \frac{\varepsilon_k^s[\Delta_k^2 + \varepsilon_k^f(\varepsilon_k^f + \varepsilon_k^s)]}{\omega_2}\right]\right\} \tag{8}$$

where $\omega_{1,2}$ are the excitation energies of the system given by (7). Changing the sum over k to an integral, we have,

$$N = \frac{k_{\rm F}^3}{4\pi^2}\int_{-\overline{\mu}}^{\infty} \sqrt{(x + \overline{\mu})}\left\{1 - \frac{x}{2(\overline{\omega}_1^2 - \overline{\omega}_2^2)} \times \left[\frac{(\overline{\omega}_1^2 - \overline{\Delta}_{\rm cf}^2 + \overline{V}^2 - \alpha x^2)(1+\alpha) - \overline{\Delta}_k^2}{\overline{\omega}_1} - \frac{(\overline{\omega}_2^2 - \overline{\Delta}_{\rm cf}^2 + \overline{V}^2 - \alpha x^2)(1+\alpha) - \overline{\Delta}_k^2}{\overline{\omega}_2}\right]\right\}{\rm d}x \tag{9}$$

where x is a dimensionless variable. The over-bar in a given quantity means that it is renormalized by the Fermi energy $E_{\rm F}$. Then, we obtain the gap equations to be solved with the number equation. These will yield the behavior of the system along the crossover from the weak coupling to the strong coupling regime. In the following we consider independently the cases of intra- and inter-band interactions.

3.1. Intra-band case

For intra-band pairing, we follow Nozières *et al* [3] and Duncan *et al* [8] and consider the interaction G being given by a separable potential

$$G(k, k') = \frac{G_0}{\{(1 + k^2/k_0^2)(1 + k'^2/k_0^2)\}^{1/2}}. \tag{10}$$

In fact, in a lattice, the potential should be a function of $\mathbf{k} - \mathbf{k}'$; $V(\mathbf{k} - \mathbf{k}')$ can be written as a sum of several components with $s, p, d\ldots$ symmetries as: $V(\mathbf{k} - \mathbf{k}') = V_s(\mathbf{k}, \mathbf{k}') + V_p(\mathbf{k}, \mathbf{k}') + V_d(\mathbf{k}, \mathbf{k}') + \cdots$ [25]. Here we are interested in extended s-wave superconductivity, thus only $V_s(\mathbf{k}, \mathbf{k}')$ couples to the superconducting s-order parameter. Moreover, in a cubic bipartite lattice $V_s(\mathbf{k}, \mathbf{k}')$ is separable, as shown by Bastide *et al* [24].

If the potential takes the form of equation (12), the intra-band superconductor order parameter is given by,

$$\Delta_k = \frac{\Delta_0}{(1 + k^2/k_0^2)^{1/2}}, \tag{11}$$

where k_0 is related to the range R_0 of the interaction ($R_0 = 1/k_0$). The coupling strength G_0 and the intra-band gap amplitude Δ_0 are constants. Finally, the gap equation is obtained from the propagator (5), and at $T = 0$ we have,

$$\frac{1}{G_0} = -\sum_k \frac{1}{2(\omega_1^2 - \omega_2^2)(1 + k^2/k_0^2)}\left[\frac{(\omega_1^2 - \varepsilon_k^{s2})}{\omega_1} - \frac{(\omega_2^2 - \varepsilon_k^{s2})}{\omega_2}\right]. \tag{12}$$

A conventional BCS superconductor has a natural cutoff to solve this equation: in this case the attractive interaction is mediated by phonons, and this attraction can be limited to energies smaller than the Debye energy ($\hbar\omega_{\rm D}$) close to $E_{\rm F}$. In the strong coupling regime this cutoff cannot be accepted since in this situation all electrons participate in the interaction, including those outside of the Debye energy shell. In this case the integral for the gap becomes divergent. The solution to eliminate this divergence is to renormalize the gap equation by introducing the s-wave scattering length a_s to describe the system interaction. a_s can be positive or negative [26]. For $\Delta_{\rm cf} = 0$, the intra-band gap equation is given by,

$$-\frac{1}{k_{\rm F}a_s} = \frac{1}{\pi}\int_{-\overline{\mu}}^{\infty} \frac{1}{[1 + (x + \overline{\mu})/\overline{E}_0]} \times \left\{\frac{\sqrt{(x + \overline{\mu})}}{(\overline{\omega}_1^2 - \overline{\omega}_2^2)}\left[\frac{(\overline{\omega}_1^2 - x^2)}{\overline{\omega}_1} - \frac{(\overline{\omega}_2^2 - x^2)}{\overline{\omega}_2}\right] - \frac{1}{\alpha\sqrt{(x + \overline{\mu})}}\right\}{\rm d}x. \tag{13}$$

Now, $1/k_{\rm F}a_s$ plays the role of a dimensionless coupling constant, where $k_{\rm F} = \sqrt{2mE_{\rm F}}$ is the Fermi wave vector. When $1/k_{\rm F}a_s \to -\infty$ one obtains the weak coupling regime (BCS limit), while $1/k_{\rm F}a_s \to +\infty$ gives the strong coupling limit (BEC limit) [26]. $\overline{E}_0$ is the energy related to the momentum $E_0 = k_0^2/2m$ and the over-bar on a given quantity means that

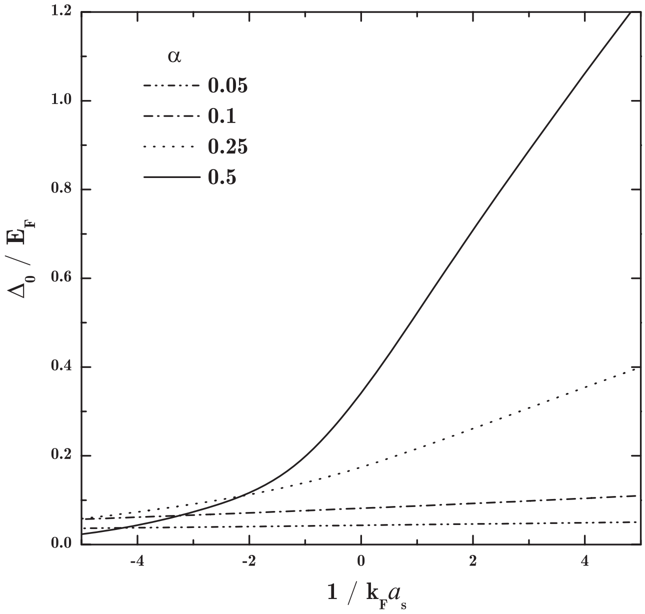

Figure 1. The zero temperature intra-band gap as a function of the interaction strength (see text) for different values of the ratio of the effective masses α.

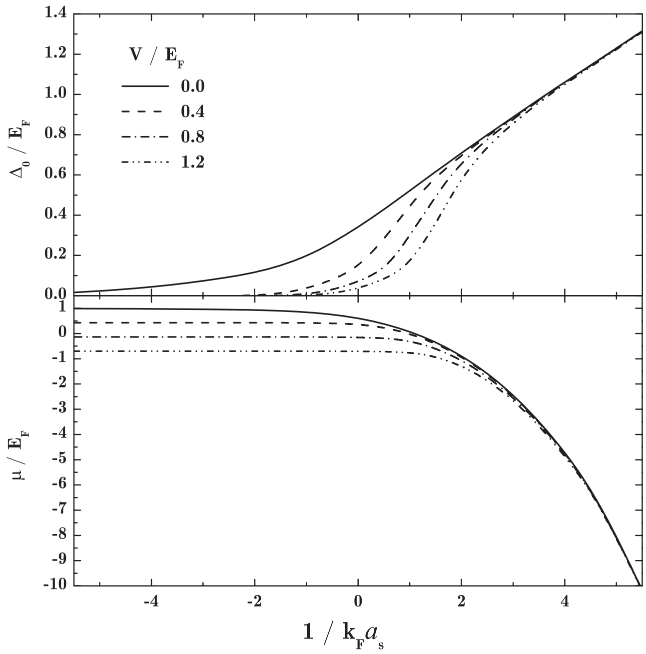

Figure 2. The zero temperature intra-band gap Δ_0, and chemical potential μ as functions of the interaction strength $1/k_F a_s$, for several values of hybridization V. The mass ratio is fixed at $\alpha = 0.5$. For $V \neq 0$, a_S must be larger than a critical value for superconductivity to appear (see figure 6).

it is normalized by E_F. The gap equation (13) must be solved simultaneously with the number equation (9).

Figure 1 shows the gap solutions for several values of the ratio of the effective masses α. One can see that as this ratio becomes very small, i.e. the quasi-particles in the narrow band become too heavy, the crossover to strong coupling only occurs for very large values of the interaction. Let us consider the case of $\alpha = 0.5$ where the BCS–BEC crossover occurs for moderate values of the coupling strength. We consider the dilute case, i.e. the case where the inter-particle spacing is larger than the interaction range, such that, $(k_0/k_F)^2 \gg 1$. This condition can also be expressed in terms of the characteristic energies, the ratio between E_0 and E_F. We choose $\overline{E}_0 = 500$, the same order of magnitude as that considered by Duncan *et al* [8]. In the BCS limit and for $V = 0$, the chemical potential practically does not differ from the Fermi energy E_F, and the superconducting gap is much smaller than E_F as expected. With increasing coupling strength the pairs become more tightly bound, the momentum distribution broadens [2, 7], and μ decreases, as shown in figure 2. The effect of increasing hybridization is also shown in this figure. For larger V, the chemical potential practically remains constant in the weak coupling regime although it is reduced with respect to E_F for all values of $V > 0$. We can see in figure 2 that the crossover from the BCS to the BEC regime remains smooth as hybridization increases. The main effect of increasing hybridization is to shift the crossover between these two regimes to stronger values of the interaction.

Figures 3 and 4 show the dispersion relations of the quasi-particle excitations above the superconducting ground state for the weak and strong coupling regimes, respectively. In the former case the dispersions are typical of those found in multi-band superconductors with a dip close to the Fermi wave vector of the band of the electrons with attractive interactions [22].

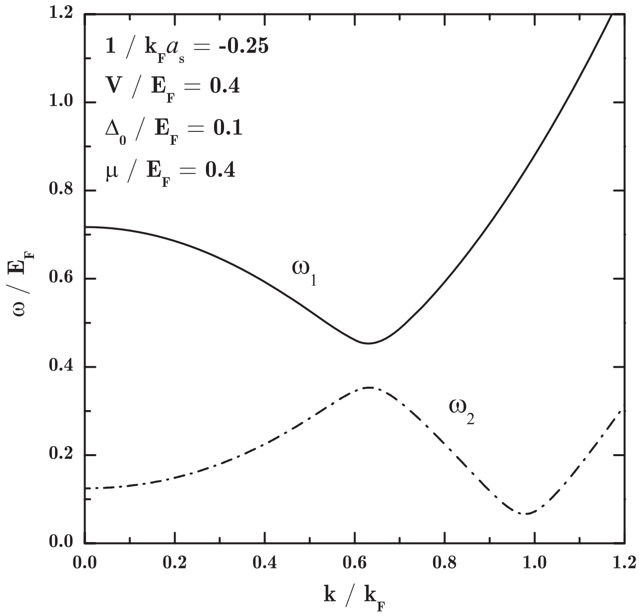

Figure 3. Dispersion relation of the excitations in the weak coupling regime [22].

In the strong coupling case shown in figure 4 the dispersion relations are similar to those of Bose particles with a quadratic dispersion. The gap at $k = 0$ is just the dissociation energy of the bosons formed by the strongly coupled pairs of fermionic quasi-particles.

It is interesting to point out that when either intra- or inter-band interactions vanish, superconducting correlations of the

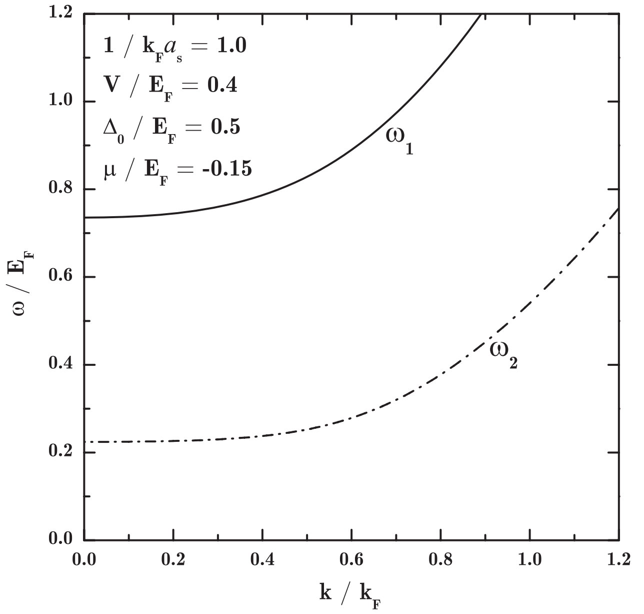

Figure 4. The dispersion relation in the strong coupling regime. The dispersions are free particle-like with a gap related to the dissociation energy.

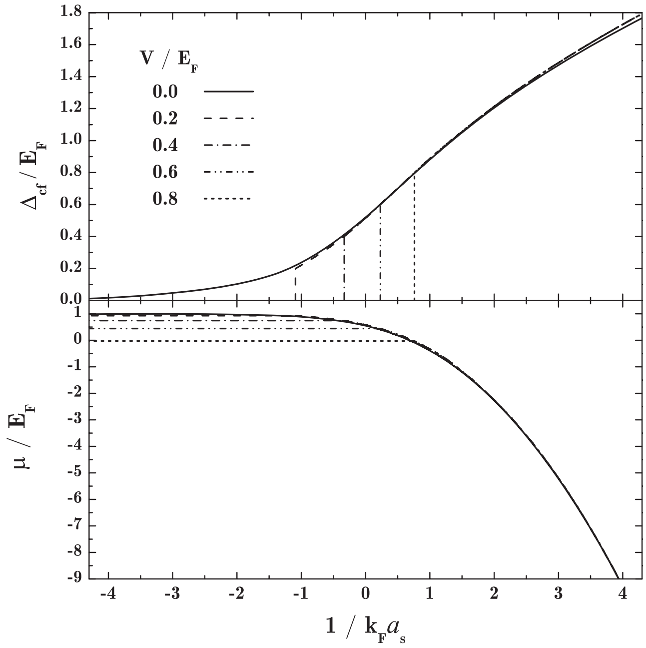

Figure 5. The inter-band gap Δ_{cf} and the chemical potential μ at $T = 0$ as functions of the dimensionless coupling $1/k_{\mathrm{F}}a_s$ for several values of hybridization V. The ratio of the masses $\alpha = 0.5$.

other type are never present in the system. This occurs in spite the fact that the anomalous Green functions involving quasi-particles of the same or different types are always different from zero: for example, if the intra-band gap is different from zero, also the inter-band anomalous Green function is different from zero, due to hybridization, but integration of this anomalous Green function leads to a vanishing inter-band order parameter. Then, both interactions should be present for the coupled inter- and intra-band problem to appear in its full complexity [22].

3.2. Inter-band case

For the inter-band case, the gap equation is obtained from the propagator equation (4), and at $T = 0$ and $\Delta_0 = 0$ we find,

$$-\frac{1}{U} = \sum_k \frac{1}{2(\omega_1^2 - \omega_2^2)} \left[\frac{(\omega_1^2 - \varepsilon_k^s \varepsilon_k^f - \Delta_{\mathrm{cf}}^2 + V^2)}{\omega_1} - \frac{(\omega_2^2 - \varepsilon_k^s \varepsilon_k^f - \Delta_{\mathrm{cf}}^2 + V^2)}{\omega_2} \right]. \quad (14)$$

Following the same procedure used to renormalize the intra-band gap equation we obtain equation (14) as

$$-\frac{1}{k_{\mathrm{F}}a_s} = \frac{1}{\pi} \int_{-\overline{\mu}}^{\infty} \left\{ \frac{\sqrt{(x+\overline{\mu})}}{(\overline{\omega}_1^2 - \overline{\omega}_2^2)} \left[\frac{(\overline{\omega}_1^2 - \alpha x^2 - \overline{\Delta}_{\mathrm{cf}}^2 + \overline{V}^2)}{\overline{\omega}_1} - \frac{(\overline{\omega}_2^2 - \alpha x^2 - \overline{\Delta}_{\mathrm{cf}}^2 + \overline{V}^2)}{\overline{\omega}_2} \right] - \frac{2}{(1+\alpha)\sqrt{(x+\overline{\mu})}} \right\} \mathrm{d}x. \quad (15)$$

The gap equation (15) should be solved simultaneously with the number equation (9) as was done previously in the intra-band case. In figure 5 we show the results of the numerical solution for the inter-band problem. Differently from the intra-band case, the zero temperature normal–superconductor transition induced by increasing the strength of the interaction in the presence of hybridization ($V > 0$) is now discontinuous or first order, even in the strong coupling limit. For $V/E_{\mathrm{F}} > 0.4$ and $\alpha = 0.5$ the transition already occurs in the strong coupling region, i.e. there is a discontinuity of the gap value, when superconductivity appears. Then, in the inter-band case for $V > 0$ the system does not present a smooth evolution as obtained in the intra-band case. This is the most significant difference between both kinds of superconductivity. Note, however, that in both cases, hybridization acts to the detriment of superconductivity. For a fixed interaction, increasing the ratio V/E_{F} eventually destroys superconductivity, either continuously (intra-band case) or discontinuously in the inter-band case. In figure 6 we plot the zero temperature critical values of hybridization V_{c} which destroy superconductivity, for both cases of intra- and inter-band interactions, as functions of the interaction strength $1/k_{\mathrm{F}}a_s$. It is clear that to suppress inter-band superconductivity a smaller value of hybridization is required than in the intra-band case, even in the strong coupling limit. Thus, the inter-band case seems to be a less stable form of superconductivity. The inter-band case with $V = 0$ and $\alpha \neq 1$ has been treated by Baranov *et al* [27]. In this case there is no SQCP.

4. Conclusions

In the present work, we have studied the crossover between the weak and strong coupling limits of superconductivity in a two-band model in a mean field approximation in the presence of hybridization. As shown by Nozières and Schmitt-Rink [3] this approximation describes quite accurately this crossover. We

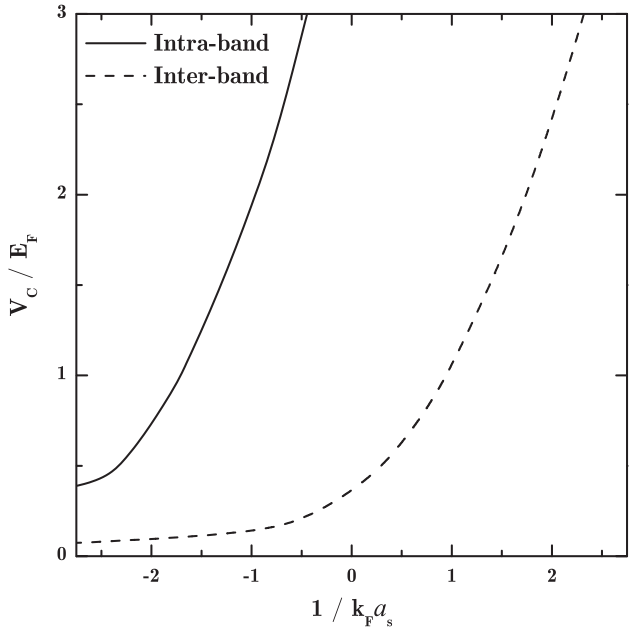

Figure 6. The critical values of the hybridization V_c at $T = 0$ as functions of the coupling strength for the intra-band and inter-band superconductivity. The ratio of the masses $\alpha = 0.5$.

obtain the normal and anomalous Green functions which are then used to determine the gap and number equations. These equations are then solved self-consistently. In order to treat the strong coupling limit and renormalize both the gap and number equations, we have to introduce scattering lengths for the two-band problem. For $V > 0$ in the intra-band case and for a dilute system we obtain a smooth evolution between weak and strong coupling limits. In the inter-band case we find that the normal–superconductor transition is first order both for increasing the interaction at a fixed value of the hybridization or for a fixed interaction and varying the hybridization. In this case there is no smooth evolution from one regime to another. In every case we find that increasing hybridization extends the weak coupling regime and the crossover to strong coupling occurs for larger values of the interactions. We also found that to suppress inter-band superconductivity a smaller value of hybridization is required than in the intra-band case, showing that the inter-band case is a less stable form of superconductivity. Rather surprisingly we have found that for a system of heavy quasi-particles, the crossover for the strong coupling Bose–Einstein regime occurs very slowly and requires very strong interactions.

What would be the effect of considering fluctuations in our two-band problem? First notice that the present model differs significantly from the usual BCS problem in one main aspect. In the latter, it is well known that any interaction induces superconductivity and consequently there is no superconductor quantum critical point associated with a normal to superconductor transition [6] at $T = 0$. As one considers the effect of temperature in this problem it becomes imperative to include fluctuations in order to have a correct description of the strong coupling limit. Otherwise the critical temperature at which superconductivity vanishes increases steadily with the coupling strength. This is non-physical and the unbounded behavior of the critical temperature is in fact associated with pair dissociation rather than with the onset of superfluidity of strongly coupled pairs. The correct T_c which describes the condensation of these pairs is only obtained when fluctuations are included [3, 6]. In our case, since we are working at zero temperature the unbounded value of the critical hybridization with the coupling strength is in fact expected, since to destroy superfluidity at $T = 0$ it is necessary to destroy the bosonic pairs, otherwise the ground state is always superfluid.

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